Internet Engineering

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Information Systems Analysis

part 1 – Petri Nets

THE DEVELOPMENT OF THE POTENTIAL AND ACADEMIC PROGRAMMES OF WROCŁAW UNIVERSITY OF TECHNOLOGY

Structure of a lesson

This approach consists of five exercises which are a part of laboratory assignments of "information technologies". The main purpose of this approach is to acquaint students with chosen models of Petri nets and their usage.

Each exercise consists of a theoretical (control exercises) and practical part (practical exercises). To be able to begin the practical part, the theory should be known to each student first. Self-study questions will be attached to each theoretical part.

There is a short instruction-information attached to each exercise, for chosen tool (software).

Above mentioned construction of exercises will help students to obtain theoretical knowledge by self- study as well as practical usage during the exercises.

Petri Nets

The Petri nets are the mathematical model which enables system description. In general the Petri nets are the places, transitions and directed arcs (from places to transitions and vice versa). There are different Petri net variants found in literature – from standard Petri nets, through timed Petri nets, stochastic nets to coloured Petri nets. Precise description of each type is beyond this instruction. Necessary for each part definitions can be found in its theoretical part.

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Instruction to the first laboratory assignment of Petri Nets

Subject: Introduction to Petri nets.

Theoretical introduction – Definition of a Petri net:

Definition 1.

A *Petri net* is a 7-tuple: $N = <P, T, F, H, W, C, M_0>$

where:
P – a set of places,
T – a set of transitions,
$F \subseteq (P \times T) \cup (T \times P)$ – a set of arcs,
$W: F \rightarrow N$ – an arc weight function (N – natural number),
$H \subseteq (T \times P)$ – a set of inhibitor arcs (definition of the weight function for inhibitor arcs
is omitted, because $W: H \rightarrow \{1\}$),
$C: P \rightarrow N$ – a place capacity function,
$M_0: P \rightarrow N$ – an initial marking function.

Example E2.1.

There are four seasons – spring, summer, autumn and winter. If the seasons are represented by the places, respectively: $p0$ – spring, $p1$- summer, $p2$- autumn, $p3$-winter then the respective season sequence can be represented by a Petri net, as shown in Fig. 2.1.

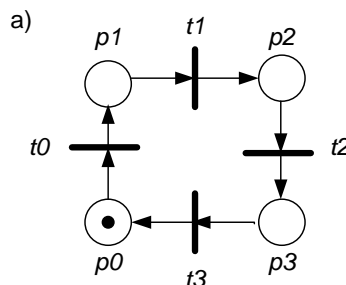*Fig.2.1. The Petri net for the sequence of seasons*

The formal notation of the PN given in Fig. 2.1. is as follows:

$P = \{p_0, p_1, p_2, p_3\}$,
$T = \{t_0, t_1, t_2, t_3\}$,
$F = \{\{p_0, t_0\} \rightarrow 1, \{p_1, t_1\} \rightarrow 1, \{p_2, t_3\} \rightarrow 1, \{p_3, t_3\} \rightarrow 1, \{t_3, p_0\} \rightarrow 1, \{t_0, p_1\} \rightarrow 1, \{t_1, p_2\} \rightarrow 1, \{t_2, p_3\} \rightarrow 1\}$,
$I = \varnothing$,
$M_0 = \{p0=1, p1=0, p2=0, p3=0\}$.

A natural number assigned to each place (e.g. $p0$=1) denotes a number of tokens in this place. The initial marking will be denoted in a short form e.g. M_0={1,0,0,0}, where the position in the set is the number of a place (first position will be numbered as 0). There are no inhibitor arcs (they will be show in next section), therefore $I = \varnothing$. If weight of some arc is equal to 1, then a label for this arc is omitted. The token shows a current season, therefore in Fig. 2.1 we have spring.

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Example E2.2.

Our customer needs a software for traffic light control system. Our job is to create a PN diagram that shows how a controller must work. The system is given in Fig.2.2.

Assumption: We have two autonomic systems (one for automobiles, one for people) and we must synchronize these systems. Examples of PNs for this system are given in Fig.2.3.
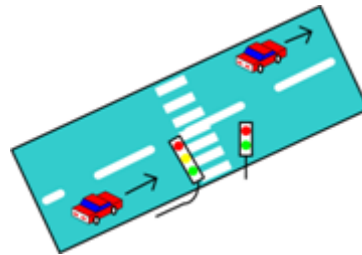

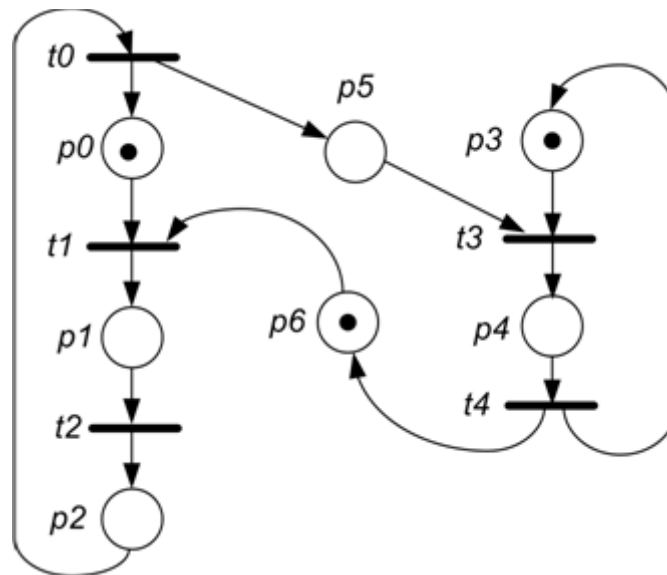***Fig.2.2.*** *Pedestrian crossing on a one-way section of a roadway*

Fig.2.3. *Synchronization of traffic lights, $M_0=\{1,0,0,1,0,0,1\}$*

The meaning of places in Fig.2.3. is the following:

- $p0, p1, p2$ – traffic lights for cars, respectively $p0$ – red, $p1$ – yellow, $p2$ – green,

- $p3, p4$ – traffic lights for pedestrian, respectively $p3$ – red, $p4$ – green,

- $p5, p6$ – places servants to the synchronization (e.g. flags placed in the software implementation).

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Theoretical introduction – states in Petri nets

Definition 2.

State S_i of Petri Net *PN* is described by marking M_i that is denoted as:

$S_i = (M_i)$, where $M_i \neq M_j$ for each $i \neq j$.

Therefore, the state S_0 is denoted by the initial marking M_0. The passage from one state to another is called a change of the PN state. This change results from firing one active transition. For the PN given by the Definition 1, the state is described by one feature only, namely the marking, for other PN definitions the marking may be insufficient.

Definition 3.

Transition t_i is *enabled* in marking M, if and only if:

- each input place contains a sufficient number of tokens,
- each input place with an inhibitory arc contains no tokens.

Therefore for the transition t_i to be fired, conditions (1) and (2) must be met.

$$(\forall p)(M(p) \geq W(p, t_i)) \tag{1}$$

$$(\forall p)((I(t_i, p) = 1) \Rightarrow (M(p) = 0)) \tag{2}$$

Definition 4.

The firing of a transition t_i consists of two steps:

- remove $W(p, t_i)$ tokens from each input place,
- add $W(t_i, p)$ tokens to each output place.

These operations are given by formula (3).

$$\forall(p)M'(p) = M(p) - W(p, t_i) + W(t_i, p) \tag{3}$$

Example E2.3.

We know that seasons change, therefore (from intuition) a PN can show only one season at a moment. Hence, we have four cases for the PN given in Fig. 2.1 (see Fig. 2.4.).
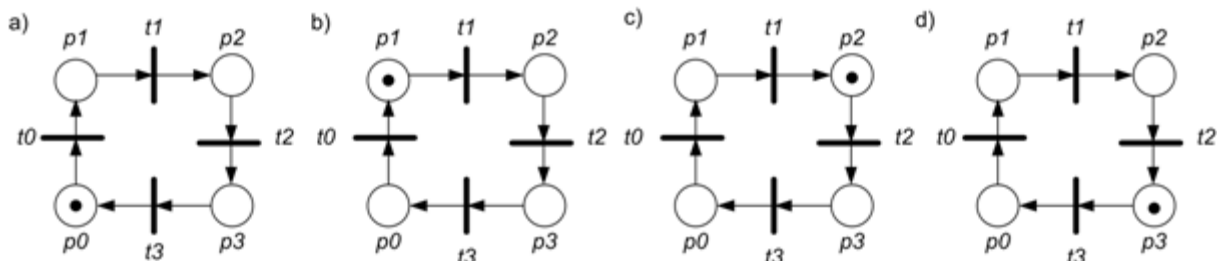

Fig. 2.4. *PN states for season change: a) initial state $S_0 = (M_0 = \{1,0,0,0\})$ – spring,*
b) state $S_1 = (M_0 = \{0,1,0,0\})$ (counted from S_0 after firing transition t_0),
c) state $S_2 = (M_0 = \{0,0,1,0\})$ (after firing transition t_1),
d) state $S_3 = (M_0 = \{0,0,0,1\})$ (after firing transition t_2)

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The change of a PN state is usually represented by a reachability graph. The reachability graph for PN from Fig. 2.3. is given in Fig. 2.5.
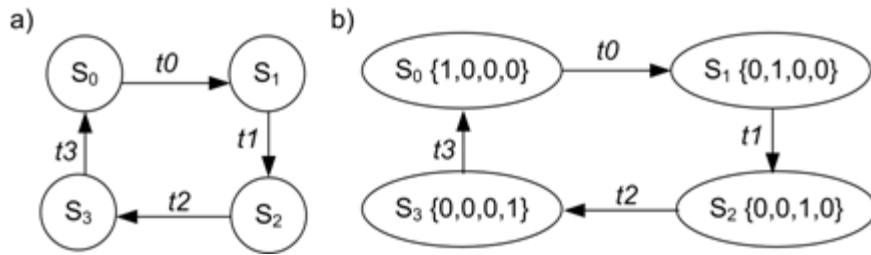

Fig. 2.5. *The reachability graphs for PN for season change*

Example E2.4.

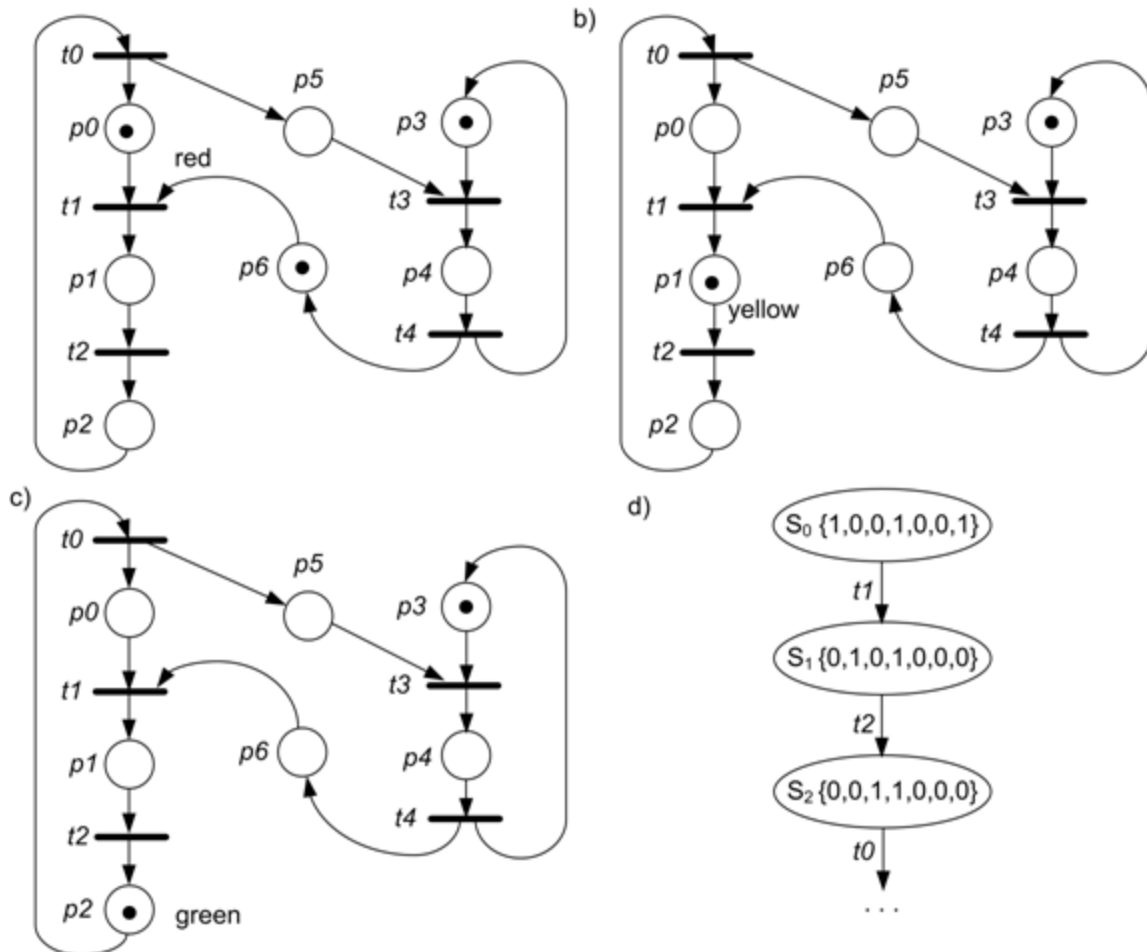The change of traffic lights until the moment in which the green lights for cars light up.

Fig.2.6. *PN for traffic light (see Fig. 2.2.):*
a) initial state (red light for cars),
b) second state, after firing the transition t_1 (yellow light for cars),
c) third state, after firing the transition t_2 (green light for cars),
d) the first three states in the reachability graph

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Now behaviour of the PN from Fig.2.6. will be shortly explained.

In Fig. 2.6a the first state of signalization is given. There is red light for cars and people. For PN from Fig. 2.6a only one transition is enabled. There is transition t1 (according to definition 1.3). The firing of this transition (according to definition 1.4) makes the following change in marking: in places p0 and p6 we subtract 1 token (according to weight of the arcs) and next we add 1 marker.

After firing transition t1 we have the PN given in Fig. 2.6b. For this PN only the t2 transition is enabled and can be fired. After each firing we have a PN given in Fig. 2.6c.

Control exercises

Exercise C2.1.

Describe the Petri nets given in Fig. 2.3 according to the Definition 1 (as shown for PN given in Fig. 2.1.).

Exercise C2.2.

Work out drawings representing the PN states in Fig. 2.6. and finish the diagram in Fig. 2.6d.

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Exercise C2.3.

Explain notions (with your own words):

- place –

- transition –

- state of Petri net –

- enabled transition –

Exercise C2.4.

What does the „firing transition" mean?

Exercise C2.5.

Work out a PN modelling the traffic light control system. If traffic light does not work, then a pulsing yellow light will be shown.

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Practical exercises

Accomplish the following tasks.

Exercise P1.1

Use laboratory software to build a PN model for the PN given in Fig. 2.6a and then to carry out simulation. After the end of simulation, please compare obtained results with the performance worked out in Exercise C2.2.

Exercise P1.2.

Work out a PN model showing changes of processes states (see Fig. 2.7.) – remembering that only one process can run at the same time.
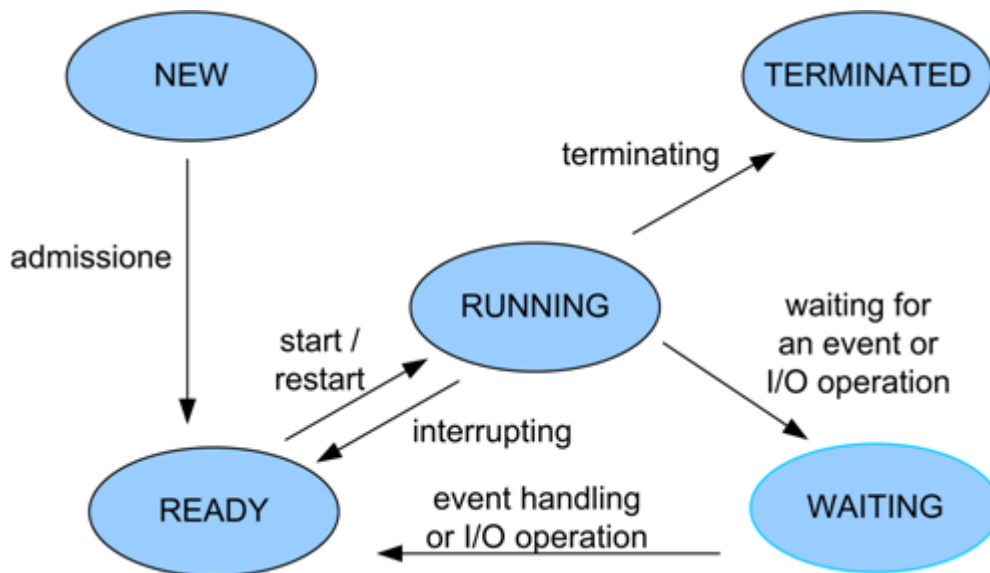

Fig. 2.7. *States of processes.*

For a PN given in Fig. 2.7. do the following:

- give a formal specification (according to the Definition 1),
- carry out simulation for maximally 10 new processes and check it in places representing running states of processes is no more than 1 token (for each state),
- build a reachability graph for PN with 2 new processes.

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Subject: Case study of using Petri nets.

Theoretical introduction – Encryption using public key:

Assumption:

It is true that: $P(T(M))=T(P(M)=M$, where:

- P – a public key,

- S – a private (secret) key,

- M – a message,

- P(M) – a message translation using public key,

- T(M) – message translation using private key.

Example E3.1.

Mr. James Bond wants to send a message M to the principal Ms. "M", therefore:

- Bond creates a message $M1=P_{principal}(M)$ and sends the message M1,

- "M" receives the message M1 and reads $M=S_{principal}(M1)$.

There is one problem in this scheme : how "M" knows the Bond is not impersonated by Dr. Kaufman?

Therefore, the communication must be in another scheme:

- Bond makes a message: $M2=S_{Bond}(M)$ and $M1=P_{principal}(M+M2)$,

- Bond sends M1,

- "M" receives M1 and has $M+M2=S_{principal}(M1)$, next reads $M'=P_{Bond}(M2)$

- M must be equal to M' (if the sender is the Bond ☺).

Attention: If Bond and "M" have not met then they should have someone "trusted" to confirm that the public keys are actually Bond's and Ms. M's (issue a certificate).

More information is given in [2], [3].

Example E3.2.

The rail-road crossing.

A crossing of rails with a car road is illustrated in Fig. 3.1. More information is given in [MS06].

When a Train is arriving the Sensor Z recognizes when it is in the section RO (between the points R and O). A signal from sensor R is sent immediately to Controller when the Train is passing the point R. Next, the Controller should initiate a light-audio signal and (after 10s) close the Gate.

The Gate is controlled by two signals: Down, Up. The Down (Up) signal causes the Gate to go down (up). If the Train is going through the railroad crossing, the Down state is a safe one.

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When Controller has received the Down Gate signal from sensor G, which indicates that the Gate is Down, then it opens semaphores.

Semaphore (S) and Auxiliary Semaphore (AS) can be in two states: Opened, Closed. They both have to be in the same state. If the Semaphore S is in the Closed state then the Train has to be stopped before it. They are controlled by the Controller using the signals: Open, Close.

When the last point of the Train is passing the point O, then the Controller closes semaphores and opens the Gate.
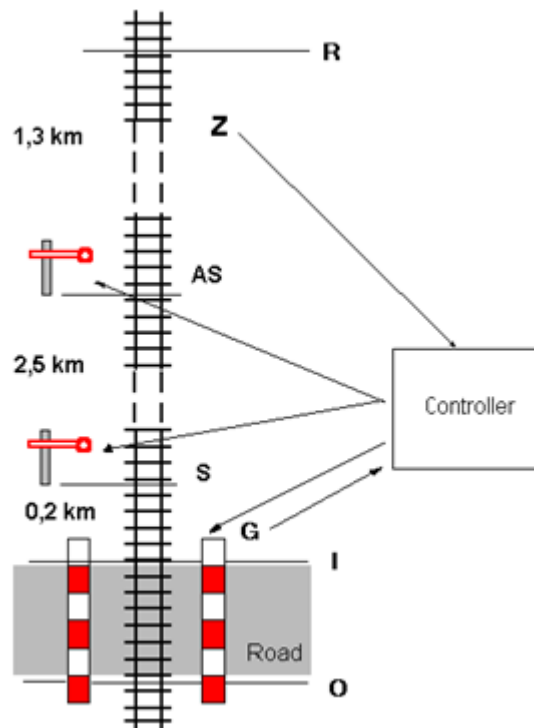

Fig. 3.1. *The simplified rail-road crossing model*

Control exercises

Exercise C3.1.

On the basis of the lecture materials, explain such behavioural properties as:

- firing sequence –

- reachability –

- liveness –

- boudedness –

- reversible net –

- coverable marking –

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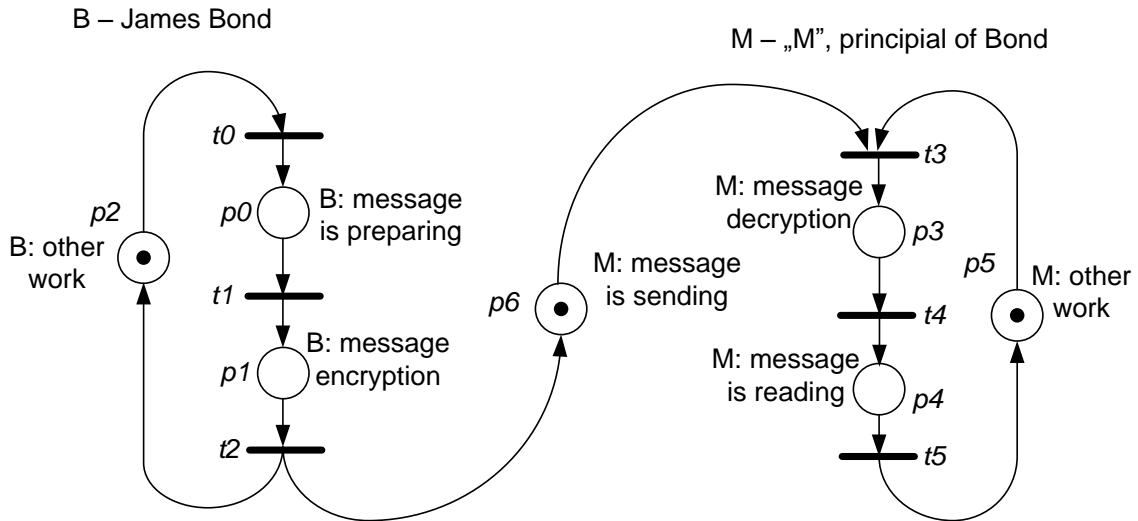
Exercise C3.2.

Fig. 3.2. *Communications Bond – „M" using public key*

For the PN for communication between Bond and "M", which is given in Fig. 3.2., do the following:

- perform analysis of behavioural properties of the PN given in Fig. 3.2.,

- prepare a formal description for PN given in Fig. 3.2 (according to definition 1),

- modify the message transfer so that a next message can be sent only after the first one has been received by „M" (the PN must be safe) and then change the description in the example 3.1 so that it corresponds to the model represented by a new PN model,

- modify the PN according the extended description.

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Exercise C3.3.

Finish the PN model given in Fig. 3.3., where the part for the Controller is omitted.
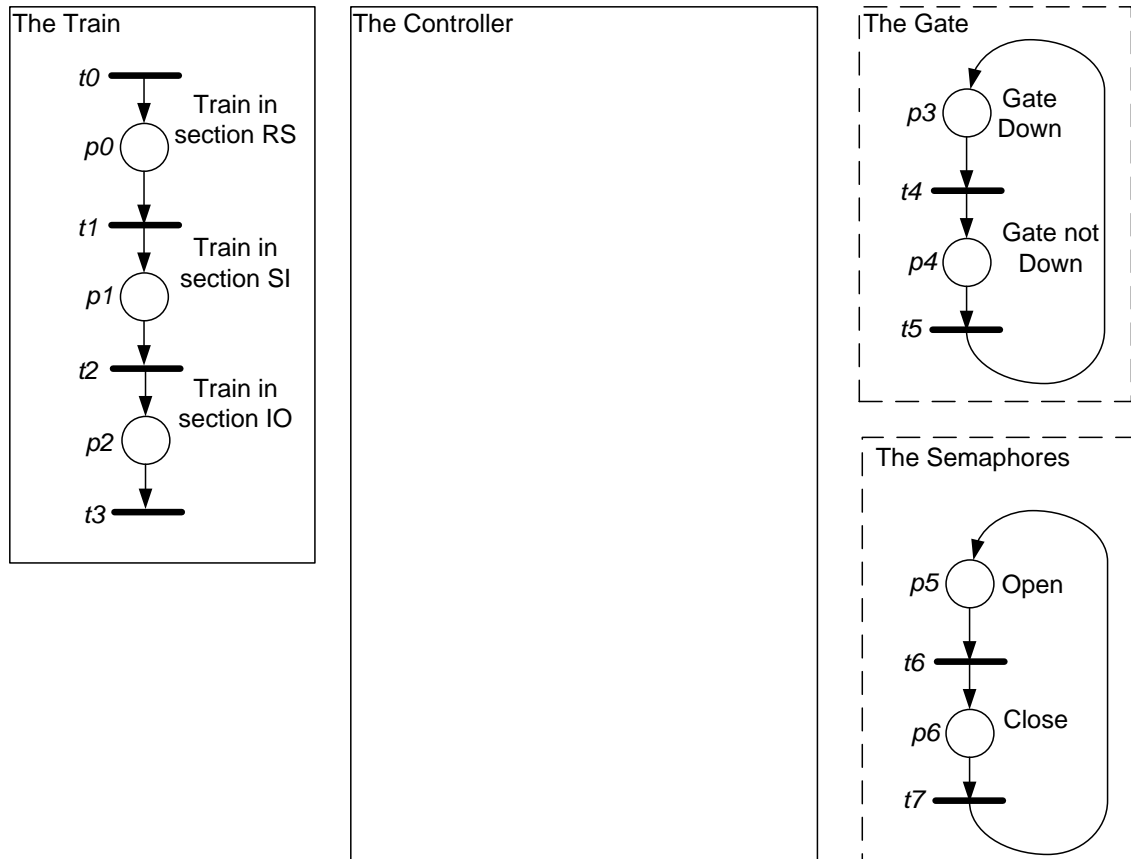

Fig. 3.3. *PN for rail-road crossing*

Practical exercises

Accomplish following tasks.

Exercise P3.1.

Analyse the PN from the exercise C3.2 (the latest model). *Attention:* The exercise C3.2 must be accomplished before!

Exercise P3.1.1.

Perform the analysis of behavioural properties.

Exercise P3.1.2.

If PN model is without inhibitors arcs, then explain a new model (with inhibitor arcs).

Exercise P3.1.3.

Modify communication in such a way that the possibility of a message loss would be taken into account.

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Exercise P3.2.

Analyse the PN from the exercise C3.3. *Attention:* the exercise must be accomplished before laboratory!

Exercise P3.2.1.

Perform the analysis of behavioural properties .

Exercise P3.2.2.

Modify the PN according to a condition: "only one train can be in section RO".

Exercise P3.2.3.

Perform the analysis of behavioural properties .

Exercise P3.2.3.

Define safety critical states (with marking modeling a situation: "train in section IO and the Gate are open"). Correct the model if the safety critical states are possible in a correctly working system.

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Instruction to the fourth and fifth laboratory assignments of Petri Nets

Subject: Timed Petri nets.

Theoretical introduction – Definition of timed Petri net:

Definition 5.

A timed *Petri net(TPN)* is 8-tuple: $N =<P, T, F, H, W, C, M_0, SI >$

where:

P – a set of places,

T – a set of transitions,

$F \subseteq (P \times T) \cup (T \times P)$ – a set of arcs,

W:$F \rightarrow N$ – arc weight function (N – natural number),

$H \subseteq (T \times P)$ – a set of inhibitor arcs (definition of the weight function for inhibitor arcs is omitted, because W:H $\rightarrow$ {1}),

C:$P \rightarrow N$ – a place capacity function,

M_0: $P \rightarrow N$ – an initial marking function,

SI: $T \rightarrow Q_+ x(Q_+ \cup \{\infty\})$ – a function assigning a static interval of firing time to each transition.

A Q_+ denotes a set of rational non negative numbers. Therefore, two numbers α^s and β^s such that $SI(t_i) = <\alpha^s, \beta^s>$ and $0 \leq \alpha^s < \infty$, $\alpha^s \leq \beta^s$, $\beta^s \leq \infty$ are assigned to every transition t_i. These numbers define a time interval in which the transition t_i has to be fired (time instant when the transition t_i becomes enabled is treated as zero time).

Example E4.1.

For timed Petri nets for the sequence of seasons, we assume that a day is a time unit. We know that: spring lasts 93 days, summer – 93 days, autumn – 90 days and winter – 89 days.

We can make a sequence of seasons using TPN model in the Fig. 4.1 (compare it with Fig. 2.1.).
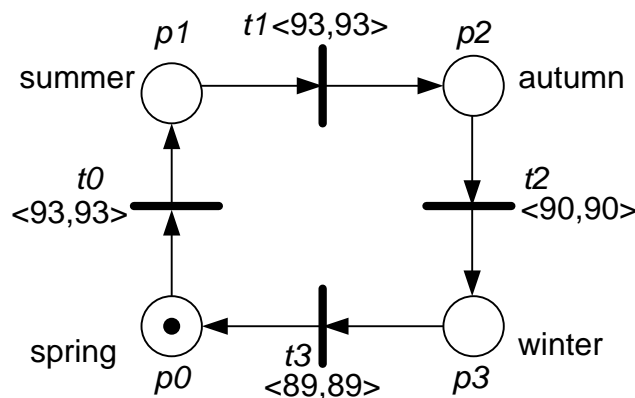

Fig.4.1. *TPN for sequence of seasons*

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Formal description of the PN given in Fig. 2.1. is as follows (compare it with the PN for sequence of seasons definition in section 2.1):

$P=\{p_0,p_1,p_2,p_3\}$,

$T=\{t_0,t_1,t_2,t_3\}$,

$F=\{\{p_0,t_0\}\to1, \{p_1,t_1\}\to1, \{p_2,t_3\}\to1, \{p_3,t_3\}\to1, \{t_3,p_0\}\to1, \{t_0,p_1\}\to1, \{t_1,p_2\}\to1, \{t_2,p_3\}\to1\}$,

$I=\varnothing$,

$M_0=\{p0=1,p1=0,p2=0,p3=0\}$,

$SI=\{t_0\to<93,93>, t_1\to<93,93>, t_2\to<90,90>, t_3\to<89,89>\}$ in short form:

$SI=\{<93,93>,<93,93>,<90,90>,<89,89>\}$, where a position in the set is the transition number.

Definition 6.

A State S of Petri Net *TPN* is described by a marking M and by a vector I – the vector of pairs of non-negative values calculated for every transition which is enabled firing in same marking.

$S_i=(M_k,I_l)$, where for each state the pair: M_k, I_l must be different.

Definition 7.

Transition t_i is *enabled* if and only if:

- each input place contains a sufficient number of tokens,

- each input places with inhibitory arcs has no token.

- is enabled for time (all other enabled for marking transitions can be firing later).

Therefore for the passage t_i conditions (4) - (6) must be fulfilled.

$$(\forall p)(M(p) \geq B(p,t_i)) \tag{4}$$

$$(\forall p)((I(t_i,p)=1) \Rightarrow (M(p)=0)) \tag{5}$$

$$\alpha_i \leq \theta \leq \min\{\beta_k\} \tag{6}$$

where:

θ – relative (to "now") time of firing the transitions t_i,

t_k – other transition enabled for marking,

k – ranges over the set of transition enabled by M.

Definition 8.

Firing of the transition t_i consists of three steps:

- removal of $W(p,t_i)$ tokens from each input places ,

- addition of $W(t_i,p)$ tokens to each output place,

- calculating a new vector I.

These operations are given by formula (7) and three steps as follows:

$$\forall(p)M'(p)=M(p)-W(p,t_i)+W(t_i,p) \tag{7}$$

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STEP 1: Remove all pairs from vector *I* that are related to transitions disabled when t_i is fired.

STEP 2: Shift times of all remaining firing intervals by the value θ.

$$\alpha_k = max\{0, \alpha_k - \theta\}, \; \beta_k = \beta_k - \theta \tag{8}$$

STEP 3: Introduce pairs of the new transition enabled.

$$\alpha_p = \alpha^S_p, \beta_p = \beta^S_p, \tag{9}$$
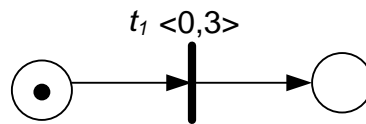
Let us analyze the example given in Fig. 4.2.

$$t_1 \; <0,3>$$

Fig.4.2. *The TPN example*

The transition t_1 is enabled to firing, but firing time θ can be e.g. 0, 1.2, 1.46, 1.2345. Hence we have infinitely many states. Therefore, we describe firing time using inequalieties, e.g. $0 \leq \theta \leq 3$. Instead of vector *I* we have now an inequality system that is called a domain and instead of one state we have a set of states (because number θ is infinity) that is called a class.

Definition 9.

State classes *C* are pairs: $\qquad C_i = (M_k, D_l)$

where:

- *M* − is the marking (all state in the class have the same marking),
- *D* − is the firing domain.

More information is given in [BM82], [BM83].

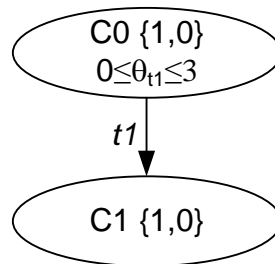The classes diagram for TPN given in Fig. 4.2 is given in Fig. 4.3.

C0 {1,0}
$0 \leq \theta_{t1} \leq 3$

t1

C1 {1,0}

Fig.4.3. *Classes for TPN from Fig. 4.2.*

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Control exercises

Exercise C4.1.

Explain notions using your own words:

- classes –

- domain –

- enabled transition (in timed Petri nets) –

Exercise C4.2.

What are the main differences between PN and TPN?

Exercise C4.3.

How many different markings may states in one class have?

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Exercise C4.4.

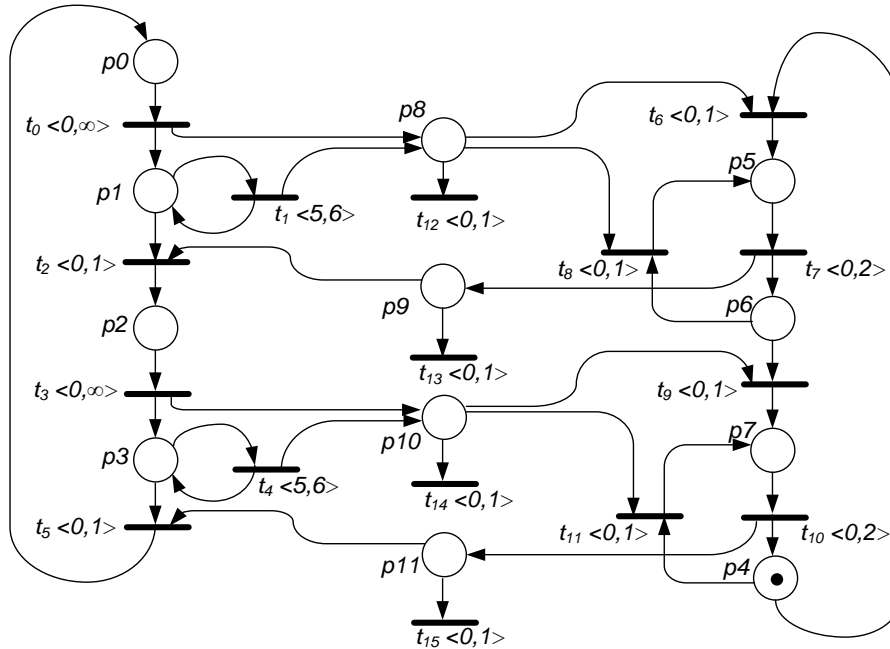In [BD91] a simple message protocol including time-out value is given, see Fig. 4.4. Work out the class diagram for this TPN.

Fig.4.4. *A simple protocol messages and including time-out value, source [BD91]*

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Practical exercises

Exercise P4.1.

Analyse the timed Petri nets given in Fig. 4.4. and compare them with results obtained in the exercise C4.4.

Exercise P4.2.

Extend the model given in Fig. 4.1. so that it takes the leap year into account.

Exercise P4.3.

Extend the model for traffic light controller according to the following conditions:

- people press a button to obtain green light,
- duration of green light for people is equal to 30 second,
- duration of green light for cars must not be shorter than 45 second.

Exercise P4.4.

On the basis of the model from the exercise **P3.2.3.**, work out your own solution with regard to losing and resending the message.

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Instruction to the sixth laboratory assignment of Petri Nets

Subject: Generalized stochastic Petri nets (GSPN).

Theoretical introduction – Definition of GSPN:

Definition 10.

GSPN is 7-tuple: $N = <P, T, F, H, \Pi, W, M_0>$

where:

P - a set of place,

T - a set of transitions; let us assume also that T_i – a sub set of immediate transitions, T_t – a sub set of timed transitions,

$F \subseteq (P \times T) \cup (T \times P)$ – a set of arcs,

$H \subseteq (T \times P)$ – a set of inhibitor arcs,

Π: $T \rightarrow N$ – priority function (N – natural number); for timed transition Π *(timed transition) =0* (lowest priority), for immediate one Π *(immediate transition)*≥1,

$W:T \rightarrow R$ – is a function that associate R (real value) to the transition:
  • for timed transitions: a (possible marking dependent) rate of negative exponential distribution specifying the firing delay,
  • for immediate transition: a (possible marking dependent) firing weight.

M_0: $P \rightarrow N$ – an initial marking function.

The main differences between definitions 1. and 10. are:

- definition of the weight function for arcs is omitted in def. 10. (it is assumed that it is equal to 1),

- definition of the place capacity function is omitted in def. 10. (no restrictions on capacity of places),

- the priority function and W function are defined for transition in def. 10.

More information can be found in the lecture or in [BLP07].

Control exercises

Exercise C5.1.

A Petri net tool Pipe 2.5 is described in [BLP07]. Please compare the 10th definition with the definition given in [BLP07] (see Figure 5.1) and explain the differences. Are the definitions congruent?

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$$GSPN = (S,T,\Pi,I,O,H,M_0,W)$$

where

- S is a set of places

- T is a set of transitions, $S \cap T = \emptyset$

- $I,O,H : T \to N$ $(N = S \cup T)$, are the input, output and inhibition functions

- $M_0 : S \longrightarrow I\!N$ is the initial marking

- $\Pi : T \longrightarrow I\!N$ is the priority function that associates the lower priorities to timed transitions and higher priorities to immediate transitions. Immediate transitions therefore always have priority over timed ones.

- $W : T \to R$ is a function that associates a real value to the transitions. $w(t)$ is:

  - a (possibly marking dependent) rate of a negative exponential distribution specifying the firing delay, when transition t is a timed transition (represented by a hollow rectangle).

  - a (possibly marking dependent) firing weight, when transition t is immediate (represented by a filled rectangle).

Fig. 5.1. GSPN definition, source [BD91]

Exercise C5.2.

Let us assume, that $t1 \in T_i$ and $t2 \in T_t$ are enable. Which one of them will be fired? Explain your answer.

Exercise C5.3.

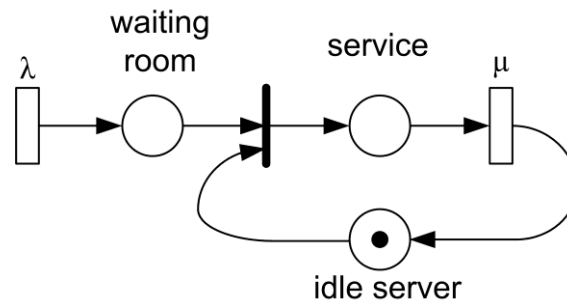Please discuss the GSPN shown in Figure 5.2. (explain the meaning of the symbols, indicate the timed transition, etc.).

Fig. 5.2. GSPN definition, source [M]

Exercise C5.4.

Explain the main differences between timed transitions in GSPN and TPN.

Exercise C5.5.

Explain: „M/M/1 queue".

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Practical exercises

Exercise P5.1.

Work out a GSPN model for handling of requests to the web server.

We have one WWW Server and one backup server. Two ways to requests handling is shown in Fig. 5.3.:

a)   handling by the primary server,

b)   handling by the primary and backup server (as hot spare).

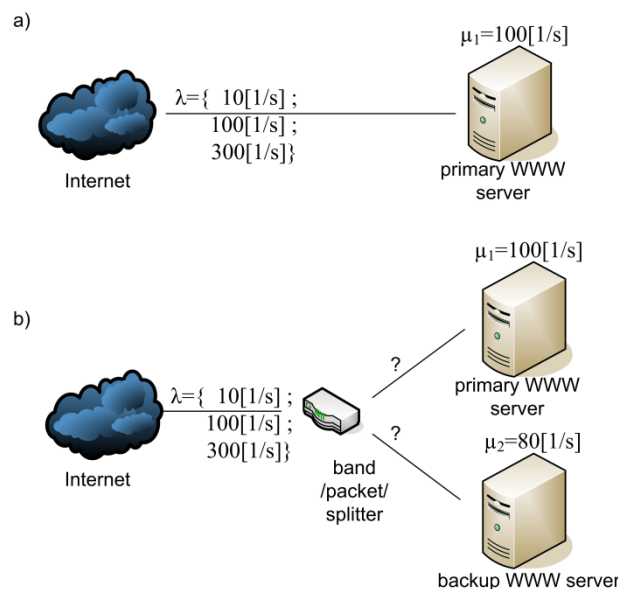Service of requests shall be in accordance with the model of M/M/1 queue.

Fig.5.3. *Request handling by a) one WWW server,*
b) two WWW servers (each server have its own queue)

Basic tasks:
- prepare GSPN model for system given in Fig. 5.1 a),
- make an analysis of GSPN for given λ and explain results.

Additional tasks:
- prepare GSPN model for system given in Fig. 5.1 b)
- specify optimal rules for band splitter (based on GPSN simulations results).

Exercise P5.2. GSPN model for a cooling system of warp-speed generator

Let us assume, that spaceship „Enterprise" („Star Trek" series) uses two independent cooling subsystems for warp-speed generator. For each subsystem, the time to failure is described by exponential random variable with parameter $\lambda=0,0001$. Repair time is described by exponential random variable with parameter $\mu=0,001$. Only one subsystem can be repaired at a time.

Prepare GSPN modeling cooling system, knowing that the spaceship is out of warp-speed due to two cooling subsystems failure. Make analysis and explain result.

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[BLP07] Bonet P., Lladó C. M., Puigjaner R., Knottenbelt W., PIPE v. 2.5: a Petri Net Tool for Performance Modeling, Palma de Mallorca, Universitat de les Illes Balears, Spain, 2007;   http://www.doc.ic.ac.uk/~wjk/publications/bonet-llado-knottenbelt-puijaner-clei-2007.pdf

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More information is given in references to the lecture.